Multi-objective Optimization and Post-optimal Analysis on Multiple Car Structure Design toward Energy Conservation and Mass-customization

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Abstract. Under sustainable development goals, multi-objective optimization has been highly notified and supported rational and flexible decision making. In the case of car industries, energy conservation and mass-customization are becoming major interests to meet such trend under global competition. To pursue this goal in a lean and agile manner, various simulation and optimization techniques are applied to real-world car design. To promote such development, one of the Japanese car companies recently released a bi-objective bench-mark problem on multiple car structure design. Since it is a large optimization problem that requires us to apply computationally tough method, in this paper, we have proposed a unique procedure incorporated with our multi-objective optimization method known as $MOON^2$ and a new method named downsizing NSGA-II. Moreover, to enhance its usefulness in practical engineering tasks, we engage in a post-optimal analysis that tries to comprehensively re-consider the prior result before the final decision. In numerical experiments, we have shown the proposed procedure can efficiently solve the original problem and move adaptively on the post-optimal analysis in the same framework. Finally, the advantage is compared with the other studies and the propose idea is shown useful toward qualified and manifold decision making.

Keywords: Multi-objective optimization, Multiple car structures design, Post-ptimal analysis, Real world benchmark problem

1. INTRODUCTION

As represented by energy and/or environmental issues, modern technologies are facing with various difficult problems incidental to sustainable development goals. To cope with such situation, multi-objective optimization has been highly notified and supported rational and flexible decision making. In the case of car industries, energy conservation and mass-customization are considered as major interests to meet such trend under global competition. To pursue this goal in a lean and agile manner, various simulation and optimization techniques have been applied to real-world car design associated with the concept of V&V (Verification & Validation, Shiratori et al., 2013). To promote such development, one of the Japanese car companies recently released a bench-mark problem on multiple car structure design in public through an academic society. It is a large complicated bi-objective optimization problem that requires us to provide a certain computationally efficient method.

Hence, in this paper, we propose a unique procedure incorporated with our multi-objective optimization method known as $MOON^2$ (Shimizu & Kawada, 2002) and a new method named downsizing NSGA-II (Yoo & Shimizu, 2018). Moreover, to enhance its usefulness in practical engineering tasks, we encourage to carry out post-optimal analysis (Shimizu, Yoo & Sakaguchi, 2016) that tries to comprehensively re-consider the prior solution against various uncertainties before the final decision. Through numerical experiments, we have shown the proposed idea can derive the prior solution efficiently and move adaptively on the post-optimal analysis in the same framework. In the numerical experiment, the advantage is shown through comparison with the other studies. From all of these, we claim the propose idea is definitely useful toward qualified and manifold decision making.

The rest of this section is organized as follows. In Section 2, we describe the framework of the proposed approach for practical decision making. Section 3 concerns with the benchmark problem and discuss on the effectiveness of the proposed approach. Some conclusions are given in Section 4.

2. PROPOSED APPROACH FOR PRACTICAL DECISION MAKING

2.1 Framework of the Proposed Idea

Generally speaking, it is quite inefficient for practical decision making just to solve the optimization problem. Actually, we need to totally engage in several processes accompanying with it. Actually, we can name value system design and problem formulation as the prior processes while post-optimal analysis as the post process. Moreover, it is essential to notice uncertain factors and/or errors encountered in each process. A framework of such idea is shown in Fig. 1 by using boxes (processes) and arrows (troublesome factors). Thereat, the troublesome factors from the upper side represent the universal one regardless of situations while those from the lower side dependent one. For example, subjective value judgement of decision maker (DM) is likely unstable and system parameters in mathematical model are substantially uncertain. Computational errors are inevitable when the algorithm is running. On the other hand, value function may occasionally be irrelevant or there happens to miss some necessary objectives or oppositely to add extra ones. It is usual to approximate non-linear model as linear one for simplicity. We cannot completely remove some gaps between the reality and its regression or response surface model. Moreover, unsuitable optimization method might be applied to the problem under concern and DM would response inconsistently on his/her preference in multi-objective optimization. Inadequate candidates could be selected at the stage of final decision or certain changes in decision environment happen to occur after the optimization.

From all of those, in practice, noting the uncertainties and errors, we should cope with every process carefully and not stick to the centered optimization process. In particular, the post-optimal analysis becomes extremely important since it has a possibility to compensate and/or dismiss every defect referring to the uncertainties of the foregoing processes. In so far studies, however, such idea has not been discussed deeply.

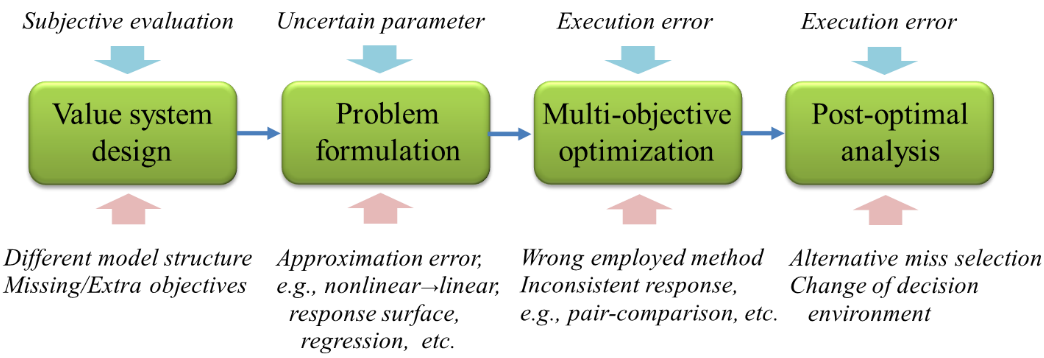

Fig.1 Essential processes for practical optimization and uncertain factors involved in its framework.

2.2 Multi-objective Optimization by Down Sizing NSGA-II Incorporated into MOON^2

In general, multi-objective optimization problem (MOP) is described as follows.

$$\text{(p.1)} \quad \text{Min} \quad \boldsymbol{f}(\boldsymbol{x}) = \{ f_1(\boldsymbol{x}), \ldots, f_N(\boldsymbol{x}) \}$$

$$\textit{subject to} \;\; \boldsymbol{x} \in X = \left\{ \boldsymbol{x} \;\middle|\; \begin{matrix} g_i(\boldsymbol{x}) \le 0, & (i = 1, \ldots, m1) \\ h_i(\boldsymbol{x}) = 0, & (i = 1, \ldots, m2) \end{matrix} \right\}$$

where $\boldsymbol{x}$ denotes a decision variable vector; X, is a feasible region; and $\boldsymbol{f}$ is an objective function vector some elements of which are incommensurable and conflict with each other. This problem aims at obtaining a unique solution known as the preferentially optimal solution through subjective preference of DM.

The methods of MOP are generally classified into *throughout* and *see-and-then* approaches. The former will solve (p.1) straightforwardly while the later attempts to reveal the trade-off relation first (*see*) and articulatee the preference after that (*then*). In this *see* stage, every multi-objective evolutionary algorithm (MOEA; Coello, 2012) seems to be effective since it can derive Pareto front readily. However, practical methods for "*then*" stage are almost unavailable presently. On the other hand, we developed a "*throughout*" method known as MOON^2 and successfully applied it to various engineering problems (For example, Shimizu, Waki & Sakaguchi, 2012).

Here, let me note MOON^2 needs to identify the value function of DM beforehand. This modeling will be carried out with a suitable artificial neural network (NN) to deal with the non-linearity commonly seen in the value function. Such NN will be trained based on the training data gathered by an AHP-like pair-wise comparison (Satty, 1980) on the DM's preference. Thus trained NN works with the reference objective values $\boldsymbol{F}^R$ supplied as a half of its inputs.

Once such value function is identified, the original (p.1) is simply solved as a single-objective problem as follows.

$$\text{(p.2)} \quad \underset{x}{\text{Max}} \quad V_{NN}(\boldsymbol{f}(\boldsymbol{x})) \;\; \textit{subject to} \;\; \mathbf{x} \in \mathrm{X}$$

To solve this problem practically and reasonably and then move on the post-optimal analysis in the same framework, we propose to apply a certain MOEA in terms of unique idea described below.

If we notice that objectives Min $V_{NN}(\boldsymbol{f}(\boldsymbol{x}))$ and Max V_{NN} $(\boldsymbol{f}(\boldsymbol{x}))$ always conflict with each other, the following problem is viewed as a bi-objective problem.

$$\text{(p.3) Max } \{V_{NN}(\boldsymbol{f}(\boldsymbol{x})), -V_{NN}(\boldsymbol{f}(\boldsymbol{x}))\} \textit{ subject to } \boldsymbol{x} \in X$$

Accordingly, we can solve (p.1) from the following simple procedures.

(1) Apply an appropriate MOEA for (p.3).

(2) Select the solution with the largest value of V_{NN} as the preferentially optimal solution of (p.1).

In the application of this MOEA, it is unnecessary to derive a widely spread distribution of Pareto front. It is enough to obtain only several candidates. This is also true for the post-optimal evolution carried out after that. To note these facts, we modified the algorithm of NSGA-II (Deb, 2000) so that the population size will decrease along with the evolution. Such algorithm whose pseudo code is given below is called as down-sizing NSGA-II.

```
    if(gen >α x gener)
{   popsize = β x popsize;
    if(popsize < minpop)   popsize = minpop;
}
```

where α and β are positive parameters (<1). And *gen*, *gener*, *popsize* and *minpop* represent current generation, its total one, population size and desired final size, respectively.

2.3 Post-optimal Analysis by Elite-induced MOEA and Summary of the Proposed Procedure

As mentioned already, we need to concern with various uncertain factors/errors for practical decision making. To cope with this by noting the specific properties of MOP, we propose a post-optimal evolution around the preferentially optimal solution. For this purpose, our elite induced multi-objective evolutionary algorithm (EI-MOEA; Shimizu, Takayama & Ohishi, 2012) is conveniently available.

The principle behind the idea of EI-MOEA is just simple and straightforward from the original MOEA. Actually, the algorithm is composed of the following two parts:

(1) Selection of some elite solutions under a certain allocation rule.

(2) Application of MOEA by incorporating the elite solutions into a set of random initial solutions.

We can expect each elite solution will induce the Pareto front in the direction toward its preexisting region. By adjusting the allocation rule (number of elites and their locations), DM is able to manipulate the final solutions so that the Pareto front will spread on a specific limited region. Moreover, due to the existence of the elites, selection pressure that might contribute to the accuracy and convergence speed is always kept at high level. This makes the algorithm powerful and computation load smaller. By the way, in the case of post-optimal analysis on MOP, it is reasonable to focus just on the preferentially optimal solution.

In summary, the proposed procedures that follow the framework shown in Fig.1 are listed below. It is also helpful for readers to refer to the case study in Section 3.

Step 1: Generate several trial solutions in the objective-space surrounded by the ideal and nadir solutions.

Step 2: Extract the preferences of the DM through pairwise comparison between every pair of the trial solutions.

Step 3: Train the neural network using the preference information obtained from the above responses. This trained network serves as a value function V_{NN} by properly selecting the reference objective values $\boldsymbol{F}^R$.

Step 4: Solve the original MOP (p.1) as the single-objective problem (p. 2) by the proposed idea.

Step 4.1: Apply the down-sizing NSGA-II to (p.3).

Step 4.2: Sort the resulting objective values in descending order and select the top as the preferentially optimal solution.

Step 5: Select the elite solutions from the preferentially optimal solution and its neighbors.

Step 6: Apply EI-MOEA (post-optimal evolution by the down-sizing NSGA-II) to derive the several candidates for the final decision.

Step 7: Move on the post-optimal analysis over the selected candidates.

Actually, Step 7 is carried out through reviewing them in decision variable space besides in the objective space and conditions of constraints as well. This must become comprehensive one including concerns not considered as pure mathematical procedures.

3. CASE STUDY

3.1 Problem Statement

In car industries, weight saving has been a major interest toward energy/material conservations. On the other hand, motorization in developing countries promotes mass-customization to meet a variety of customer demands while reducing development and production costs. To pursue these goals in a lean and agile manner, it is required to provide some method for the parts design commonly available among the multiple car structures. In real-world car design, however, it is widely known these two goals conflict with each other.

Noticing those facts, we convinced our idea is just amenable for this resolution and can demonstrate it through a benchmark problem released recently. Though the outline of this problem is described below, more information is available from the literature (Kohira et al., 2017) and web site (URL, 2017)

Objective functions:
(1) Minimize the total weight of three designs (denoted as CDW, SUV, C5H hereinafter)
(2) Maximize the number of common thickness parts over the three designs

Here, in the first objective, each weight is modeled by the multiple regression equation after normalizing it by the respective standard. In the second, if the difference of plate thickness at the corresponding part is less than 0.05 over the all designs, it is admitted as commonly available.

Constraints: These conditions refer to the popular requirements on car structure design such as rigidity of vehicle body, low frequency vibration and collision performance. They are given totally as 42 inequality equations (14 per each design). Actually, they are the response surface models described by the radial basis function with 1158, 1215 and 1271 centers for each, respectively. Another 12 inequality equations (4 per each) give the size relations among the decision variables. Besides these, each decision variable has a box condition (upper and lower bounds).

Decision variables: These are composed of plate thicknesses of parts over three designs and come to totally 222 (74 per each).

After all, it is a large complicated bi-objective optimization problem composed of 222 decision variables and 54 constraints described as the regression and the response surface models. Accordingly, it should be emphasized the resulting problem becomes extremely tough in optimization.

3.2 Procedures to Obtain Preferentially Optimal Solution

Since subjective information on the DM's preference is essential for MOP, we assume a virtual DM whose value function is given as Eq.(1) for convenience.

$$U(\boldsymbol{f}(\boldsymbol{x})) = \left\{ \sum_k^N w_k \left(\frac{f_k(\boldsymbol{x}) - F_k^{nad}}{F_k^{utp} - F_k^{nad}} \right)^t \right\}^{1/t} \quad (1)$$

where F_k^{utp}, F_k^{nad} and w_k denote a utopia, a nadir and a weight representing relative importance of k-th objective, respectively. And, t is a norm parameter of the value function. Hence, $U(\boldsymbol{f}(\boldsymbol{x}))$ represents the ratio of relative attainability from the utopia and takes 1.0 for the utopia and 0.0 for the nadir. In terms of this value function, we can consistently decide any reply of the virtual DM on preference.

Now, we are ready for deriving the preferentially optimal solution through the earlier part of the proposed approach. In **Step 1**, we set references (utopia and nadir) and 4 trials as shown in the margin of Table 1. The result of **Step 2** for the virtual DM ($w_1 = 0.3$, $w_2 = 0.7$, $t=1$ in Eq.(1)) is shown as a pair comparison matrix whose element a_{ij} denotes the converted value from the linguistic reply on preference of $\boldsymbol{F}^i$ over $\boldsymbol{F}^j$. That is, if $\boldsymbol{F}^i$ is "*equally*" preferable to $\boldsymbol{F}^j$, a_{ij}=1, if "*moderately*" a_{ij} =3, if "*strongly*", a_{ij} =5, if "*demonstrably*", a_{ij} =7 and if "*extremely*", a_{ij} =9. To reduce the response load, relation $a_{ji} = 1/a_{ij}$ is assumed as AHP. In **Step 3**, train NN (numbers of node for input $\{\boldsymbol{F}^i, \boldsymbol{F}^j\} = 2N = 4$, hidden = 10 and output a_{ij} = 1) using the preference data imbedded in Table 1. Thus trained NN is available as the value function V_{NN} by selecting the reference at $\boldsymbol{F}^{\mathrm{R}} = (0.25, 0.25)$. Since so far procedures are a part of MOON[2], refer to the original reference (Shimizu and Kawada, 2002) more in detail.

Now, in **Step 4**, first, solve (p.3) by the down-sizing NSGA-II under the conditions such as *popsize*=300, *gener*=1500, α=0.4, β= 0.98, *minpop* = 0.2 x *popsize*. Moreover, tuning parameters for crossover distribution index, cross-over probability, mutation distribution index and mutation probability are set at 10.0, 0.75, 50.0, 0.125, respectively and selection strategy obeys tournament rule. Then, we obtained the preferentially optimal solution such as V_{NN}= 032304 at (f_1, f_2) = (39, 2.922245) by 237172 total evaluations. In this computation, it took 11h 16m working time via a personal computer like Toshiba KIRA (Intel® Core™ i5-420U CPU@1.6GHz, Ram 8GB).

Table 1 Pair comparison matrix from Eq.(1) and objective values of reference and 4 trial ($\boldsymbol{F}^1$ - $\boldsymbol{F}^4$) solutions

	$\boldsymbol{F}^{\mathrm{utp}}$	$\boldsymbol{F}^{\mathrm{nad}}$	$\boldsymbol{F}^1$	$\boldsymbol{F}^2$	$\boldsymbol{F}^3$	$\boldsymbol{F}^4$
$\boldsymbol{F}^{\mathrm{utp}}$	1	9	4	5	3	8
$\boldsymbol{F}^{nad}$	1/9	1	1/6	1/5	1/7	1/2
$\boldsymbol{F}^1$	1/4	6	1	3	1	5
$\boldsymbol{F}^2$	1/5	5	1/3	1	1/3	3
$\boldsymbol{F}^3$	1/3	7	1	3	1	6
$\boldsymbol{F}^4$	1/8	2	1/5	1/3	1/6	1

$\boldsymbol{F}^{\mathrm{utp}}$=(70, 2.0), $\boldsymbol{F}^{\mathrm{nad}}$=(0, 4.0), $\boldsymbol{F}^1$=(68, 2.932), $\boldsymbol{F}^2$=(55, 3.362), $\boldsymbol{F}^3$=(47, 2.538), $\boldsymbol{F}_4$=(30, 3.938)

3.3 Comparison of Results by Post-optimal Evolution

In this subsection, we show the results of the post-optimal evolution taken place in the later part, i.e., Step 5 & 6. In **Step 5**, we choose 6 elite solutions that correspond to 10% of the population size. They are composed of the tripled preferentially optimal solution and other 3 neighbor solutions around it.

In **Step 6**, letting the initial and final population sizes as 60 and 20, respectively, we applied the down-sizing NSGA-II in the elite induced mode. By 100 generations with total 40420 evaluations, we derived several candidate solutions supplied to the post-optimal analysis.

So far results are shown in Fig.2 with the other results for comparison. They are illustrated together on the counter map of the present value function, i.e., Eq.(1). Thereat, "Initial design" and "Isight Optimization" denote the results by the actual (human) engineers and the commercial software named "Isight", respectively. They are reported from the car company that served the benchmark problem. On the other hand, "MOON2 (before optimal)" refers to the proposed idea (result at Step 4). Meanwhile, "postopt-MOON2" describes the Pareto front obtained following the post-optimal evolution by EI-NSGA-II (result at Step 5 & 6). Moreover, for reference, the Pareto front obtained by the ordinal NSGA-II with the condition such as population size=300, generation=1000 and evaluation number=300000 is shown as "NSGA-II".

Comparing "MOON2" with "Initial design", we know the former results apparently outperform the later. Moreover, though "postopt-MOON2" aims at deriving a local Parent front just around "MOON2", its distribution is better than the approaches obtained from "Isight" and "NSGA-II" that aim at global distribution. Particularly speaking, we can claim it is very convenient for every user since major knowledge to have here is just about NSGA-II.

3.4 A Few Examples of Post-optimal Analysis

The post-optimal analysis in **Step 7** was taken place through reviewing both "MOON2" and "postopt-MOON2" more in detail. For example, concerns should be extended to the decision variable space and the conditions of constraints. This must become comprehensive one including the dealings impossible as pure mathematical approaches. Presently, though the common thickness parts between two designs are ignored at first and concern on the tightness of the constraints is hard to involve into the problem formulation, we might make a better final decision by taking those factors into account.

To work with this concern, we selected the top five solutions having greater V_{NN} value as the candidates since they spread sparsely with each other. As shown in Table 2, they all outperformed "MOON2" (number in [ ] denotes the order of V_{NN}). This means the post-optimal evolution is also useful for improving the prior solution. Now, we compare the common thickness parts over two designs among the candidates and "MOON2". Then, second and third place Candidate #1 and #3 realize greater numbers (12) than the first place Candidate #2 (10) and "MOON2" (10) though they are a bit inferior to Candidate #2 regarding V_{NN}.

On the other hand, in Table 3, we compared the tightness of the constraints $g_i(\boldsymbol{x}) \geq 0$ (i=1,…,54). There, "Tight" column denotes the number of tight constraints ($g_i(\boldsymbol{x})$<0.05) and the remaining columns statistics of the $g_i(\boldsymbol{x})$ value. Generally, the smaller this value is, the more robust it is against uncertainties and/or changes of situations due to the larger allowance till the boundary. Among the candidates, the number of tight constraints of Candidate #1 is smallest and the other statistics (Max, Average and Variance) are well balanced.

Considering those facts that are hard to discuss in the process of the prior optimization, Candidate #1 has a high potential to be selected as the best decision after this post-optimal analysis. Moreover, the final decision thus made could be far superior to the human decision made empirically ("Initial design" shown in Fig.2). Through those discussions, we can definitely claim the significance of the proposed approach.

4. CONCLUSION

To resolve many difficult problems incidental to modern technologies, multi-objective optimization has been widely applied so far. In this study, we focused our attention on car industries and engaged in solving a practical bi-objective optimization problem associated with energy conservation and mass-customization. Since the resulting problem becomes extremely tough and requires us computationally efficient method, we have proposed a unique application of MOEA and developed a method incorporated with our multi-objective optimization method known as $MOON^2$ and a new method named downsizing NSGA-II. Moreover, to enhance its usefulness in practical engineering tasks, we have engaged in the post-optimal analysis associated with our elite-induced MOEA (EI-NSGA-II) by noticing various uncertainties and/or errors.

In numerical experiments, we have shown the proposed procedure can derive the prior solution efficiently and move adaptively on the post-optimal analysis in the same framework. The advantages of the proposed idea are verified in numerical experiments through comparison with the other studies. Finally, we claim the proposed framework makes multi-objective optimization more promising tool toward recent qualified and manifold decision making.

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Table 2 Comparison of common thickness parts over two designs among the candidates and "MOON2"

Candidate	V_{NN} (f_1, f_2)	Common thickness number 2 out of 3			
		Total	CDW /SUV	CDW /C5H	SUV /C5H
#1 [3rd]*	0.33053 (2.909, 39)	12	4	6	2
#2 [1st]	0.33708 (2.869, 37)	10	3	4	3
#3 [2nd]	0.33057 (2.866, 36)	12	4	4	4
#4 [4th]	0.33043 (2.895, 38)	9	2	5	2
#5 [5th]	0.32902 (2.855, 35)	16	3	8	5
"MOON2" [6th]	032304 (2.922, 39)	10	3	7	0

* Order as of the magnitude of V_{NN}

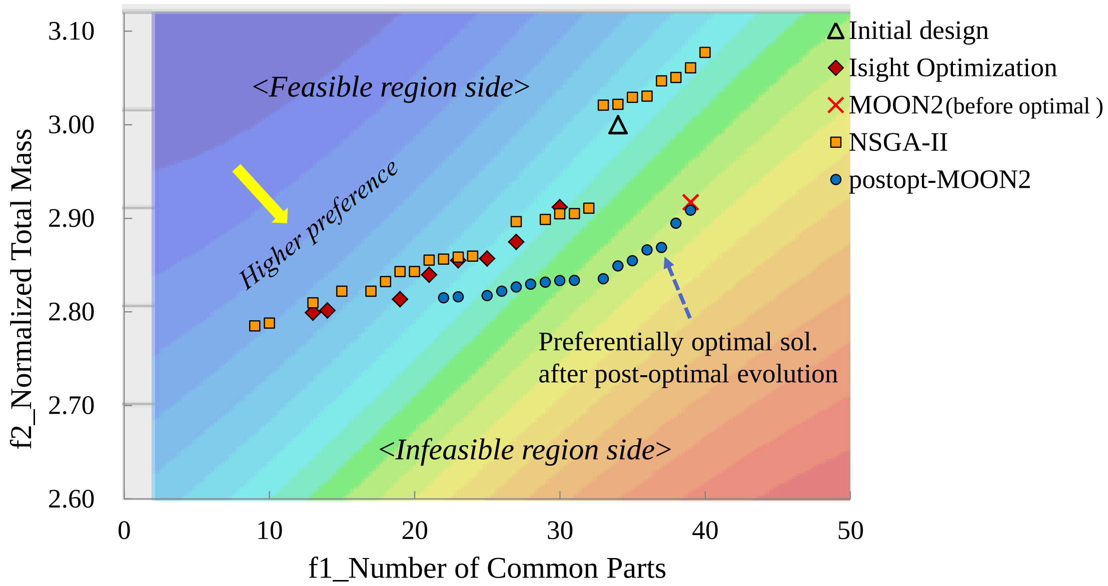

Fig.2 Comparison with other methods: "MOON2 (before optimal)" outperforms the empirical "Initial design". Moreover, "postopt-MOON2" can derive several solutions better than "MOON2 (before optimal)". It also outperforms the ordinal NSGA-II and commercial software "Isight Optimization" as the quality of Pareto front.

Table 3 Feature of the candidates on the tightness of constraints (number in [ ] denotes the order of V_{NN} value.)

Candidate	Tight number				Max			Average			Variance		
	Total	CDW	SUV	C5H	CDW	SUV	C5H	CDW	SUV	C5H	CDW	SUV	C5H
#1 [3rd]	19	6	7	6	0.418	0.379	0.372	0.106	0.108	0.119	0.013	0.014	0.012
#2 [1st]	24	10	6	8	0.418	0.327	0.391	0.098	0.096	0.107	0.014	0.009	0.014
#3 [2nd]	23	10	6	7	0.424	0.332	0.391	0.097	0.095	0.107	0.014	0.009	0.014
#4 [4th]	20	9	6	5	0.410	0.321	0.399	0.101	0.100	0.117	0.013	0.010	0.013
#5 [5th]	25	10	7	8	0.430	0.291	0.369	0.097	0.087	0.099	0.013	0.008	0.011
MOON2 [6th]	18	7	6	5	0.349	0.355	0.390	0.099	0.108	0.121	0.009	0.011	0.012

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